LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.C.A. DEGREE EXAMINATION – **MATHS & STATS**

SECOND SEMESTER – **APRIL 2013**

MT/ST 2902 – PROBABILITY AND STOCHASTIC PROCESSES

Date : 30/04/2013 Dept. No. Max. : 100 Marks
Time : 9:00 - 12:00

Section A

Answer all questions. **(10x2=20)**

1. Define a continuous random variable.

2. Define conditional distributions.

3. Define Geometric distribution.

4. A storage depot contains 10 machines, 4 of which are defective. A company selects 5 of these machines at random thinking that they are in working condition. What is the probability that all 5 of the machines are non-defective?

5. Define probability measure.

6. Define an n-Step transition probability matrix

7. When do you say a Markov chain is irreducible?

8. Define the stationary probability distribution of a Markov chain

9. State the postulates of Pure Birth Process

10. Define a Brownian motion process.

Section B

Answer any FIVE questions. **(5x8=40)**

11. State and prove addition theorem of probability for n events.

12. Derive the mean & variance of Poisson distribution.

13. A random variable X has the distribution function F,

$$F(x)=\begin{cases} o & x<0 \\ 1/2 & x=0 \\ 1/2 + x/2 & 0<x<1 \\ 1 & x\geq 1 \end{cases}$$

Show that F is neither continuous nor discontinuous. Also evaluate P (0 ≤ X ≤½).

14. If t is any positive real number of p(x) = $e^{-t}\left(1-e^{-t}\right)^{x-1}$, then find expectation of X and variance of X.

15. Explain the canonical representation of a TPM(transition probability matrix).

16. Show that the states of a two dimensional random walk are recurrent.

17. Explain in detail any two examples of renewal processes.

18. Derive Kolmogorov-Backward differential equations for birth and death process.

Section C

Answer any TWO questions. **(20x2=40)**

19. (a) Let X be a continuous random variable with p.d.f. given by:

$$f(x)=\begin{cases} k\,x, & 0 \le x < 1 \\ k, & 1 \le x < 2 \\ -kx+3k, & 2 \le x < 3 \\ 0, & elsewhere \end{cases}$$

(i) Determine the constant k,

(ii) Determine F(x), the c.d.f

(iii) If x_1, x_2 and x_3 are three independent observations from X, what is the probability that exactly one of these three numbers is larger than 1.5?

(b) State and prove Bayes theorem.

20. (a) The joint p.d.f. of two random variables X and Y is given by:

$$\frac{9(1+x+y)}{2(1+x)^4(1+y)^4};\ 0 \le x, y \le \infty$$

Find the marginal distribution of X and Y, and the conditional distribution of Y for X=x.

(b) State and prove the basic limit theorem of Markov chains.

21. (a) Show that the states of a one dimensional random walk are recurrent if p=q=1/2

(b) A Markov chain on states {1, 2, 3, 4, 5, 6} has transition probability matrix P. Find all equivalence classes and period of states. Also check for the recurrence of the states.

$$P = \begin{bmatrix} 0.5 & 0 & 0 & 0 & 0.5 & 0 \\ 0 & 0.2 & 0.2 & 0.5 & 0.1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0.6 & 0 & 0.4 & 0 & 0 \\ 0.3 & 0 & 0 & 0 & 0.7 & 0 \\ 0.2 & 0.2 & 0.1 & 0 & 0.2 & 0.3 \end{bmatrix}$$

22. Let {X(t)} be a process satisfying all the postulates of a pure birth process, then derive the transition distribution of the process.

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